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The enigma of volatility: Exploring asymmetric threshold effects in U.S. bond futures prices during yield curve inversions

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ABSTRACT

In contrast to the majority of literature that focuses on financial product volatility during financial crises, this paper stands as the first study delving into the asymmetric threshold effects on the volatility of intermediate-term and long-term U.S. Treasury bond futures prices during the inverted yield curve period.

We present compelling evidence confirming the presence of both TAR (Threshold AutoRegressive) and MTAR (Momentum Threshold AutoRegressive) effects within the sample period. Using a synchronous grid search algorithm, we simultaneously searched for the optimal threshold values for TAR and MTAR models. Our empirical findings indicate a reduction in the volatility of U.S. bond futures prices during the period of yield curve inversion. Moreover, negative shocks trigger the TAR and MTAR threshold effects, leading to an increase in the volatility of bond futures prices. Furthermore, our research has revealed that the total effects of TAR and MTAR models display contrasting correlations in response to market shocks. Consequently, if the magnitude of market shocks changes exceeds the threshold level, the influence of the TAR threshold effect could be offset by the MTAR effect. As a result, the determination of threshold values plays a significant role and simultaneously reflects the volatility sensitivity of bond futures. Based on the findings of this study regarding asymmetric volatility and sensitivity comparisons, the optimal U.S. Treasury futures options trading strategies during periods of yield curve inversion are to purchase 30-year Treasury bond futures put options or to buy 10-year Treasury bond futures call options.

This study investigates the varied responses of bond futures volatility to market shocks and assesses the sensitivity of volatility to these shocks by analyzing how impacts are transmitted through threshold values and their economic implications.

Keywords:

Yield curve inversion, Asymmetric volatility of bond futures, Volatility sensitivity of bond futures, SMTAR-GARCH model.

1 | Introduction

The yield curve inversion is a unique financial condition and macroeconomic environment. Under normal market conditions and term preference theory, long-term bonds typically have higher term premiums, leading to a positively sloped bond yield curve. Fama (1984) finds a positive relationship between term premiums, security maturity, and interest rate volatility. However, a negative slope in the yield curve holds significant implications for the financial market. Historically, when the yield on the 10-year U.S. Treasury note falls below that of the 2-year U.S. Treasury note, it reliably signals an impending recession in the U.S. economy.

It is noteworthy that the presence of an inverted yield curve does not necessarily signify that the economy is undergoing a recession or facing a financial crisis. Thus, under these unique market conditions, the primary motivation of this study is to explore the characteristics of bond price volatility, as well as its sensitivity and response to market shocks. Furthermore, previous literature has frequently highlighted the presence of asymmetric volatility effects in equities, and bonds also exhibit asymmetric volatility in response to positive or negative market news. For instance, Asymmetric volatility, first introduced by Black (1976), indicates that positive and negative news impact the conditional volatility of financial assets differently. DeGoeij and Marquering (2002) found that macroeconomic announcement shocks significantly explain this asymmetry in U.S. Treasury bond markets. Additionally, Cai and Jiang (2008) found that bond volatility can serve as a predictor of future bond returns and associated risks, making a deep understanding of its characteristics valuable for practical applications. Therefore, a crucial focus of this research is to investigate whether the asymmetric volatility effects in bonds persist under the specific condition of a yield curve inversion or if the interaction with the yield curve inversion results in different responses in the asymmetric threshold effects of bonds.

The reasons for a negative yield curve are diverse, including pessimistic economic growth outlooks, changes in inflation expectations, and aggressive central bank actions. Investigating the asymmetric volatility of bond futures during these periods can offer insights for trading strategies and derivative pricing. This area remains largely unexplored in existing literature.

This research aims to examine the presence of an asymmetric threshold effect in the volatility of U.S. medium to long-term bond futures prices during periods of inverted yield curve in the U.S. Treasury bond market. Using a Synchronous momentum TAR/MTAR GARCH (SMTAR-GARCH) model, the study investigates the existence of this effect by concurrently estimating both the Threshold AutoRegressive (TAR) model, introduced by Tong and Lim in 1980, and the Momentum Threshold AutoRegressive (MTAR) model, initially proposed by Enders and Granger in 1998. To facilitate this analysis, a novel programming algorithm is introduced to simulate two random thresholds for both TAR and MTAR components in the equations of mean and variance simultaneously. Additionally, the study explores variations in the sensitivity of 10-year and 30-year U.S. Treasury futures prices to positive and negative market news.

The empirical study analyzed 10-year and 30-year U.S. Treasury futures prices from June 1990 to December 2023. Yield curve inversions were identified using the negative yield spread between 10-year and 2-year U.S.

Treasury notes. Findings show a positive correlation between the yield curve's slope and the volatility of both 10-year and 30-year U.S. Treasury futures prices during yield curve inversions. In essence, when the slope of yield curve flattens or turns negative, bond futures price volatility tends to decrease.

To investigate the presence of asymmetry in bond futures price volatility, this study employs the Threshold AutoRegressive (TAR) and Momentum Threshold AutoRegressive (MTAR) models, using the residuals from these models to represent unexpected market shocks. If the residuals fall below the threshold (i.e., $\varepsilon_{t-1} < \text{threshold}$, indicating negative market shocks) and result in increased volatility, while residuals above the threshold (indicating positive market shocks) lead to decreased volatility, it suggests the presence of a TAR effect. Furthermore, in the MTAR model, the changes in residuals are used to capture the extent of changes in the magnitude of market shocks. If the residual changes fall below the threshold (i.e., $\Delta\varepsilon_{t-1} < \text{threshold}$, indicating more severe negative market shocks) and lead to increased volatility, while residual changes above the threshold (indicating more severe positive market shocks) result in decreased volatility, this indicates the presence of an MTAR effect.

The empirical findings reveal that during inverted yield curve periods, the volatility of 10-year and 30-year U.S. Treasury bond futures prices exhibits an asymmetric Threshold AutoRegressive (TAR) effect. Adverse events in the bond market with a significant negative impact surpassing a specific threshold level tend to increase bond futures price volatility, while positive impacts decrease volatility. Furthermore, during yield curve inversions, intensified negative market shocks and magnitudes exceeding a certain threshold lead to increased bond futures price volatility, demonstrating the existence of the Momentum Threshold AutoRegressive (MTAR) effect.

Another interesting finding emerges when considering the TAR and MTAR threshold values during a period of yield curve inversion, the two futures price volatilities display varying levels of sensitivity and responsiveness to market disturbances.

The main contribution of this paper lies in its first exploration of asymmetric threshold effects on volatility in U.S. medium- to long-term bond futures prices during yield curve inversion periods. Our empirical findings reveal both TAR and MTAR threshold effects during this phase, along with notable differences in the sensitivity and response of 10-year and 30-year U.S. Treasury futures prices to market shocks. The subsequent sections of this article are structured as follows: Section 2 elaborates on the data and methodology, Section 3 delves into the model and presents empirical outcomes, and finally, Section 4 discusses the economic significance and draws conclusions.

2 | Data and methodology

2.1 Literature and hypotheses

Fama and French (1989) posited that economic conditions have predictive power over stock and bond returns, and extensive empirical research demonstrates that the slope of the yield curve is closely related to economic activity.

Estrella and Hardouvelis (1991) were the first to statistically document this relationship in the United States. According to Howell (2018), the spread between the 10-year Treasury note and the 3-month Treasury bill is frequently used as a benchmark due to its robustness over time. However, some U.S. researchers prefer alternative shorter-term market rates, while Fama (1990) emphasizes the 5-year minus 1-year Treasury note spread. In practice, many analysts and investors lean towards the 10-year minus 2-year Treasury note spread. This preference is largely due to the 2-year Treasury note's higher liquidity and its better reflection of market expectations compared to the 3-month Treasury bill yield. According to Benenson (2022), an inversion of the yield curve, particularly the spread between the 10-year and 2-year U.S. Treasury notes, is a reliable indicator of impending recessions, although it does not specify the exact timing.

Christiansen and Lund (2002) recently investigated the correlation between interest rate volatility and yield curve shape. Their analysis indicates that interest rate volatility has a greater impact on the curvature of the yield curve than its slope. The study explores theoretical arguments regarding the interaction between interest rate volatility and yield curve steepness. For instance, the term-structure and mean reversion models tell us that bond prices are obtained as

$$P(t,T) = E_t^Q\left[e^{-\int_t^T r_s ds}\right] \tag{1}$$

$$dr_t = \{\mu(r_t,X_t) - \lambda(r_t,X_t)\sigma(r_t,X_t)\}d_t + \sigma(r_t,X_t)dW_t^Q \tag{2}$$

Where $P(t,T)$ denote the price at time t of a discount bond maturing at time T, t ≤ T, with unit maturity value, and r_t is the instantaneous spot short-rate. Drift term includes a long-term mean parameter, defined as $\mu(r_t,X_t)$, and $\lambda(r_t,X_t)$ is the market price of short-rate risk(X_t are other state variables). $\sigma(r_t,X_t)$ is the short-rate volatility.

The yield curve $R(t,\pi) = -\log P(t,t+\pi)/\pi$ is given by:

$$R(t,\pi) = r_t + \frac{1}{2}(\mu - \lambda\sigma)\pi - \frac{1}{6}\sigma^2\pi^2 \tag{3}$$

The slope of yield curve $R(t,\pi)$ is related to the short-rate volatility σ:

$$R(t,\pi_2) - R(t,\pi_1) = (\mu - \lambda\sigma)(\pi_2 - \pi_1) - \frac{1}{6}\sigma^2(\pi_2^2 - \pi_1^2) \tag{4}$$

For the reasonable parameter values, the first term dominates, except $\pi_2 - \pi_1$ is large. We expect the positive relationship between the slope of yield curve and volatility which reflects the risk premium effect of volatility, i.e., $\lambda<0$.

Based on equation (4), our initial investigation examines whether the volatilities of 10-Year U.S. Treasury note futures prices and 30-Year Treasury bond futures prices decline as the yield curve flattens, especially during periods of inverted yield curve. The first hypothesis is as follows:

H₁: The volatilities of intermediate-term (10-year T-Note) and long-term (30-year T-Bond) futures prices are smaller during the period of inverted yield curve (i.e., when the yield on the 10-year Treasury note falls below the yield on the 2-year Treasury note)

than the period when yield curve is upward sloping.

This study focuses on the volatility of bond futures prices. Previous literature has shown that bond volatility can further predict future bond returns and bond risk. Therefore, understanding the characteristics of bond volatility is valuable for practical applications in trading strategies or hedging strategy formulation. For instance, Cai and Jiang (2008) indicate that bond volatility is a significant predictor of future bond returns, underscoring the importance of understanding volatility for practical applications. Christiansen and Savva, (2023) analyze the risk-return trade-off for U.S. long-term government bonds. Their research highlights the significant impact of bond volatility on bond returns, with a distinct differentiation between “good volatility” and “bad volatility.” Specifically, the study finds that good volatility has a positive effect on bond returns, while bad volatility exerts a negative influence. These findings suggest that the nature of volatility plays a crucial role in determining the direction and magnitude of bond returns.

Additionally, related research has indicated that macroeconomic variables or changes in the yield curve slope can influence bond volatility. Huang *et al.* (2008) propose that macroeconomic factors, particularly real and monetary factors, have been shown to impact Treasury bond return volatility across different maturities. Hautsch and Ou (2008) study the predictability of U.S. government bond excess returns using yield curve factors as well as yield volatility components. They find that the slope and curvature yield factors explain up to 36% of the variation in future yearly bond excess returns, revealing the same predictability as the return-forecasting factor proposed by Cochrane and Piazzesi (2005).

Viceira (2012) studied the relationship between bond risk, bond return volatility, and the term structure of interest rates, indicating that a robust stylized fact in empirical finance is that the spread between the yields on long- and short-term bonds positively forecasts future excess returns on bonds at varying horizons. This study presents evidence that movements in both the short-term nominal interest rate and the yield spread are positively related to changes in subsequent realized bond risk and bond return volatility, particularly the intercept and slope of the yield curve, which are themselves variables that proxy for business conditions.

Moreover, the concept of asymmetric volatility, introduced by Black (1976), suggests that positive and negative news have differing impacts on the conditional volatility of financial assets. This phenomenon has received significant attention in academic literature, with studies exploring how macroeconomic news affects bond market volatility. Li and Engle (1998) studied the impact of macroeconomic announcements, such as the Producer Price Index and employment situation, on U.S. Treasury bond futures volatility using a filtered GARCH model. Their findings showed compelling evidence of asymmetric responses to positive and negative news, with bond futures volatility reacting more strongly to negative news. Positive shocks tended to decrease volatility over subsequent days, while negative shocks increased volatility.

Balduzzi *et al.* (2001) utilized intraday data from the interdealer government bond market to investigate the impact of scheduled macroeconomic announcements on prices, trading volume, and bid-ask spreads. They discovered that 17 public news releases, measured by the surprise in announced quantities, significantly influence the prices of various instruments: three-month bills, two-year notes, 10-year bonds, and 30-year bonds. The effects of these announcements vary significantly depending on the maturity of the bonds. Moreover, they found that public news releases explain a substantial portion of the price volatility observed after these announcements.

DeGoeij and Marquering (2006) examined the effects of macroeconomic news announcements on the conditional volatility of U.S. Treasury bond returns, finding that such announcements have a significant impact on bond market volatility. Using daily returns on 1-, 3-, 5-, and 10-year U.S. Treasury bonds, they found empirical evidence that macroeconomic pre-announcements raise the level of conditional bond market volatility to a great extent. Announcement shocks are less persistent than regular shocks, which suggests that the bond market incorporates the implications of macroeconomic announcement news more quickly than other information.

Beber and Brandt (2010) analyzed how bond returns and volatility responded to macroeconomic news during economic expansions and recessions. They found that macroeconomic announcements had the greatest impact when conveying negative news during expansions, with a notable but lesser effect when the news was positive during economic contractions. Their study emphasized the importance of considering the content of news and the business cycle phase. Moreover, Defond and Zhang (2014) found that asymmetries in bondholders' payoff functions play a crucial role in shaping the valuation of accounting information within the bond market.

In summary, these research studies offer valuable insights into the phenomenon of asymmetric volatility and shed light on how macroeconomic news affects volatility within the bond market.

Additionally, the advent of sophisticated econometric models has markedly improved the ability to measure and examine asymmetric volatility. Nelson (1991) made a pivotal contribution with the introduction of the Exponential GARCH (EGARCH) model. Unlike conventional GARCH models, EGARCH incorporates a logarithmic transformation, allowing it to account for the asymmetric effects of positive and negative returns on volatility. Similarly, Glosten *et al.*(1993) developed the GJR-GARCH model, which explicitly includes an asymmetric component to capture the increased volatility typically observed following negative returns. Engle and Ng (1993) made significant contributions by introducing News Impact Curves (NICs), a concept designed to visualize how new information affects future market volatility. Their research confirms that news is a significant driver of market volatility and highlights that negative news typically leads to a larger increase in volatility compared to positive news, a phenomenon commonly known as the "leverage effect." Moreover, they compare several volatility models, including GARCH, EGARCH, and asymmetric models such as GJR-GARCH, to evaluate their effectiveness in capturing the impact of news. They recognize the usefulness of the GJR-GARCH model in modeling asymmetric responses to market shocks.

Various GARCH-type models have been widely applied in studying financial asset volatility. Among these, the GJR-GARCH model has garnered substantial empirical support for its effectiveness in modeling asymmetric volatility effects. Unlike traditional GARCH-type models, this study introduces an innovative stochastic threshold GARCH model (Synchronous momentum

TAR/MTAR-GARCH), which utilizes a grid search algorithm for estimating the random thresholds. The empirical analysis will assess whether this stochastic threshold model surpasses the GJR-GARCH model in effectively capturing asymmetric volatility effects.

In this paper, we utilize the Threshold AutoRegressive (TAR) and Momentum Threshold AutoRegressive (MTAR) models, widely used for analyzing asymmetric patterns in time-series data. TAR model is an advanced nonlinear time series model that includes two or more branches based on a threshold variable. It was first introduced by Tong in 1978 and extensively detailed by Tong and Lim (1980), and it captures asymmetric effects from past residual shocks, while the MTAR model, suggested first by Enders and Granger (1998), captures residual difference effects for measuring the incremental momentum during highly volatile periods. We combine these models with the Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) model to examine volatility asymmetry in U.S. Treasury futures prices.

The main aim of this study is to investigate the asymmetric responses of bond futures prices to positive and negative market news. While the downward slope of the yield curve can signal pessimistic market expectations and extreme conditions, it is important and doesn’t necessarily indicate an economic recession. We propose hypotheses based on existing literature and test them using GARCH, GJR-GARCH, and Synchronous Momentum TAR/MTAR (SMTAR-GARCH) models. Additionally, we recommend using bond price changes instead of returns for better model performance and volatility forecast accuracy. Bollerslev *et al.* (1994) indicate that models using price changes, especially within GARCH frameworks, show enhanced predictive power for volatility when compared to return-based models. This is due to the ability of price changes to capture the persistent volatility clustering and leverage effects seen in financial time series.

H_2: The volatilities of intermediate-term and long-term bond futures prices are greater whenever the lagged residual falls below its threshold during the period of inverted yield curve (TAR effect).

H_3: The volatilities of intermediate-term and long-term bond futures prices are greater whenever the first differencing of lagged residual falls below its momentum threshold during the period of inverted yield curve (MTAR effect).

2.2 Data

In this study, we collected daily data on 10-year and 30-year U.S. Treasury futures prices from CME Group via Bloomberg. Additionally, we obtained daily data on 2-year and 10-year Treasury note yields from Bloomberg to identify instances of yield curve inversion. Our dataset covers the period from June 26, 1990, to December 29, 2023, providing an extensive timeframe for analysis. Table 1 outlines the frequency of yield curve inversions between 10-year and 2-year U.S. Treasury notes throughout our sample period. Notably, the data reveals that the yield curve inverted in response to Federal Reserve interest rate hikes in 2006 but remained unaffected during the subsequent financial crisis in 2007. This distinctive observation sets our study apart from others focusing primarily on financial product fluctuations during the crisis period.

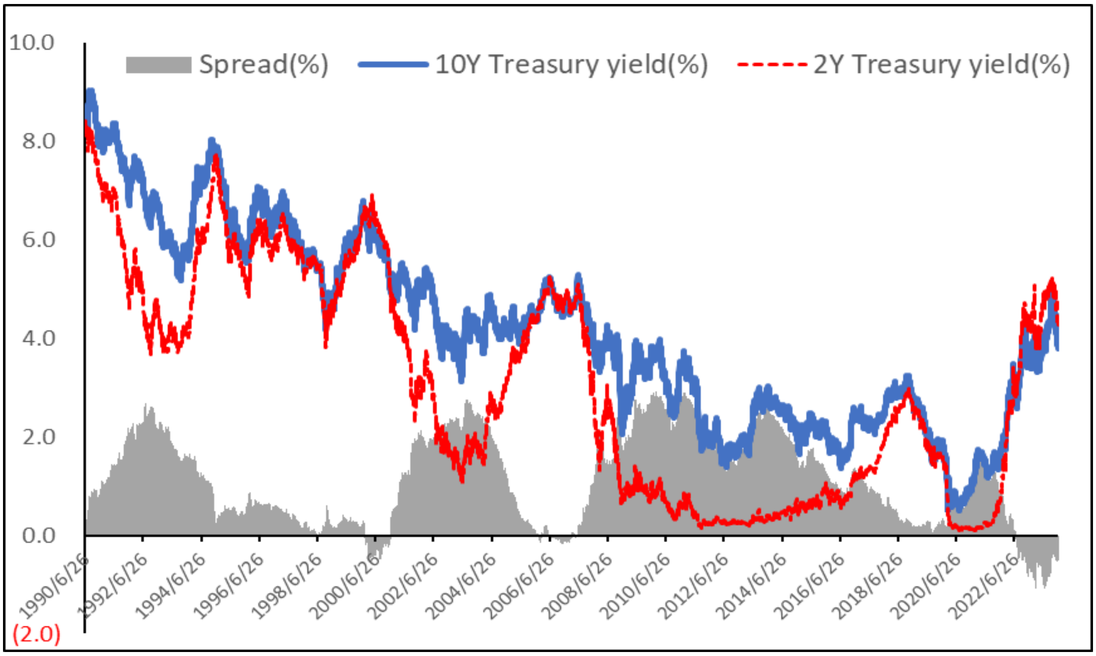

Figure 1 Daily Treasury Yields, 1990/6/26–2023/12/29

Table 1 Segmented periods of yield curve inversion from 26 June 1990 to 29 December 2023

	Period		**Trading days**
Inversion of the U.S. 10-year and U.S. 2-year Treasury note yields	26-May-98	24-Jul-98	27
	2-Feb-00	28-Dec-00	235
	27-Dec-05	30-Dec-05	3
	2-Jan-06	29-Dec-06	171
	2-Jan-07	5-Jun-07	78
	26-Aug-19	2-Sep-19	6
	1-Apr-22	29-Dec-23	380
Overall observed time	26-Jun-90	29-Dec-23	900

Note: we obtained the daily yields of 2-year and 10-year U.S. Treasury notes from Bloomberg.

2.3 Traditional TAR and MTAR models

The Threshold AutoRegressive (TAR) model is an advanced autoregressive model that incorporates two or more branches dependent on a threshold variable. This permits the model to exhibit asymmetrical behavior that cannot be explained by a single ARMA model.

$$Y_t = \alpha + \beta X_t + u_t \tag{5}$$

$$\Delta u_t = I_t \rho_1 u_{t-1} + (1 - I_t)\rho_2 u_{t-1} + \sum_{i=1}^{k} \gamma_i \Delta u_{t-i} + \varepsilon_t \tag{6}$$

where Y_t and X_t are co-integrated variables, ε_t is a white noise residual term, and ρ_1, ρ_2, and γ_i represent regression coefficients. Finally, I_t is an indicator function such that τ_1 is an unknown threshold to be simulated:

$$I_t = \begin{cases} 1 & if\ u_{t-1} \geq \tau_1 \\ 0 & if\ u_{t-1} < \tau_1 \end{cases} \tag{7}$$

Momentum Threshold AutoRegressive (MTAR) model is a type of time series model that incorporates nonlinearities and threshold effects. MTAR models are used to capture complex dynamics in time series data that cannot be adequately modeled using traditional linear AutoRegressive (AR) models. In an MTAR model, the relationship between the current value of a time series and its past values is modeled as a combination of a linear autoregressive component and a nonlinear threshold component. The nonlinear threshold component captures the idea that the relationship between the current value and past values of the time series can change depending on the magnitude of the past values. Specifically, the threshold component specifies a range of values within which changes in the time series are amplified or dampened. The MTAR model, which utilizes first differencing of the residuals as an alternative adjustment process, can be expressed as follows:

$$\Delta u_t = M_t \rho_1 u_{t-1} + (1 - M_t)\rho_2 u_{t-1} + \sum_{i=1}^{k} \gamma_i \Delta u_{t-i} + \varepsilon_t \tag{8}$$

$$M_t = \begin{cases} 1 & if\ \Delta u_{t-1} \geq \tau_2 \\ 0 & if\ \Delta u_{t-1} < \tau_2 \end{cases} \tag{9}$$

The conventional practice in estimating econometric models involves the separate estimation of Equations (5) and (6), as well as Equations (5) and (8). However, a synchronous Threshold AutoRegressive (TAR) and Momentum Threshold AutoRegressive (MTAR) model is proposed in the subsequent section, which combines the equations mentioned above.

2.4 The empirical SMTAR-GARCH

The mean and variance equations for the SMTAR-GARCH (Synchronous Momentum TAR/MTAR-GARCH) model are represented by equations (10) and (11), respectively. In equation (10), the variable $\mathrm{I_t}$ signifies the threshold dummy variable for the TAR effect.

When the residual in the mean equation exceeds the threshold value τ_1, it indicates positive news in the bond market; conversely, if the residual is less than τ_1, it indicates negative news, leading to a drop in bond prices. Hence, coefficient δ_2 measures the threshold effect of the volatility of U.S. Treasury futures prices in the face of good or bad news in the market during the inverted yield curve period.

Additionally, variable $\mathrm{M_t}$ serves as a threshold dummy variable for the MTAR effect, utilizing the differenced lagged residual to measure the asymmetric threshold effect of changes in the magnitude of market impact on U.S. Treasury futures price volatility. When the differenced lagged residual falls below the threshold value ($\Delta\varepsilon_{t-1} < \tau_2$), it indicates an intensification of the negative impact. The coefficient δ_3 is used to assess how changes in the impact of positive or negative news during the inverted yield curve period affect the volatility of U.S. Treasury futures prices. In addition, variable $\mathrm{D_t}$ represents an indicator for the yield curve inversion between the 10-year and 2-year U.S. Treasury notes.

$$R_t = \mu_0 + \mu_1 R_{t-1} + \mu_2 I_t + \mu_3 M_t + \mu_4 D_t + \mu_5 D_t \cdot I_t + \mu_6 D_t \cdot M_t + \varepsilon_t \tag{10}$$

$$h_t = \alpha_0 + \beta_1 h_{t-1} + \alpha_1 \varepsilon_{t-1}^2 + \rho_1 I_t + \rho_2 M_t + \delta_1 D_t + \delta_2 D_t \cdot I_t + \delta_3 D_t \cdot M_t \tag{11}$$

Where

$$I_t = \begin{cases} 1 \; if \; \varepsilon_{t-1} \geq \tau_{1,} \\ 0 \; if \; \varepsilon_{t-1} < \tau_1, \end{cases}$$

$$M_t = \begin{cases} 1 \; if \; \Delta\varepsilon_{t-1} \geq \tau_{2,} \\ 0 \; if \; \Delta\varepsilon_{t-1} < \tau_2, \end{cases}$$

$$D_t = \begin{cases} 1, if \; inversion \; between \; 10-year \; and \; 2-year \; U.S. Treasury \; notes \\ 0, otherwise \end{cases}$$

$\varepsilon_t = \sqrt{h_t} v_t$ is the residual of the time series, $v_t \sim^{i.i.d.} (0,1)$ is the white-noise process. $\alpha_0 \geq 0$, $\beta_1 \geq 0, \alpha_1 \geq 0, 0 \leq \beta_1 + \alpha_1 + \rho_1 + \rho_2 + \delta_1 + \delta_2 + \delta_3 < 1$.

The combination of GARCH with TAR and MTAR models can further capture the heteroscedasticity and asymmetric threshold effects in the volatility of U.S. Treasury bond futures prices. It should be noted that if the values of τ_1 and τ_2 are both set to zero, Equations (10) and (11) become similar to the GJR-GARCH mode with $\tau_1 = \tau_2 = 0$. To test H_1, we analyze if 10-year and 30-year U.S. Treasury bond futures prices have lower volatility during yield curve inversion (i.e., $\delta_1 < 0$). For H_2 (TAR effect), we check if medium- to long-term U.S. Treasury bond futures prices show higher volatility (i.e., $\delta_2 < 0$) during inversion when facing negative news shocks (i.e., $\varepsilon_{t-1} < \tau_1$). Testing H_3 (MTAR Effect), we assess if bond futures prices exhibit increased volatility (i.e., $\delta_3 < 0$) during inversion when the first differencing of the lagged residual falls below its momentum threshold (i.e., $\Delta\varepsilon_{t-1} < \tau_2$). Finally, for H_4, we use the Wald test to examine whether the total MTAR effect of market news impact during the inverted yield curve period is greater than the total TAR effect($i.e., \; \rho_2 + \delta_3 > \rho_1 + \delta_2$).

2.5 The SMTAR-GARCH estimation algorithm

In this study, we modified Chan's (1993) method for identifying a single optimal threshold in either the TAR or MTAR model.

Specifically, we developed a new grid search algorithm to estimate two random thresholds simultaneously, addressing the limitation of Chan's approach where closely estimated residuals may lead to over-simulation. This study provides a valuable contribution to the literature on threshold autoregressive models and offers a more robust method for estimating optimal thresholds in practice.

To conduct a grid search for the optimal TAR and MTAR random threshold values, it is necessary to determine the range of values for the residuals. To accomplish this, we first set both SMTAR-GARCH threshold values to zero and estimate the GJR-GARCH model using equations (10) and (11), saving the residual series ε_t . We then define the maximum and minimum values of the residual series as maxE and minE, respectively, and define the maximum and minimum values of the differenced residual series $\Delta\varepsilon_t$ as maxDE and minDE. Through this definition of extreme values in the residual series, we obtain the distribution range of residuals. To execute the grid search procedure, we need to construct a grid structure for all possible combinations of the two random threshold values. We define the incremental values for each threshold value in the grid search procedure as INC1 and INC2, representing the increment values for τ_1 and τ_2, respectively. We define INC1 as (maxE - minE) / K and INC2 as (maxDE - minDE) / K, where K is the number of times the nested structure of the grid search is divided. We then utilize a loop structure in the RATS econometrics software to search for the optimal threshold values. This is done using a nested loop that iterates over all possible combinations of threshold values.

Finally, we developed a searching algorithm using a two-layered do-loop procedure and implemented it using the RATS econometrics software. The first layer of the procedure controls the threshold increments, with the increments being narrowed down to half distances on each subsequent run. The second layer of the procedure simulate the optimal thresholds(τ_1, τ_2) for the TAR and MTAR models based on the minimum values of the Akaike Information Criterion (AIC) and Schwarz's Bayesian Criterion (SBC), as well as the maximum log-likelihood. Figure 2 illustrates the grid structure searching algorithm.

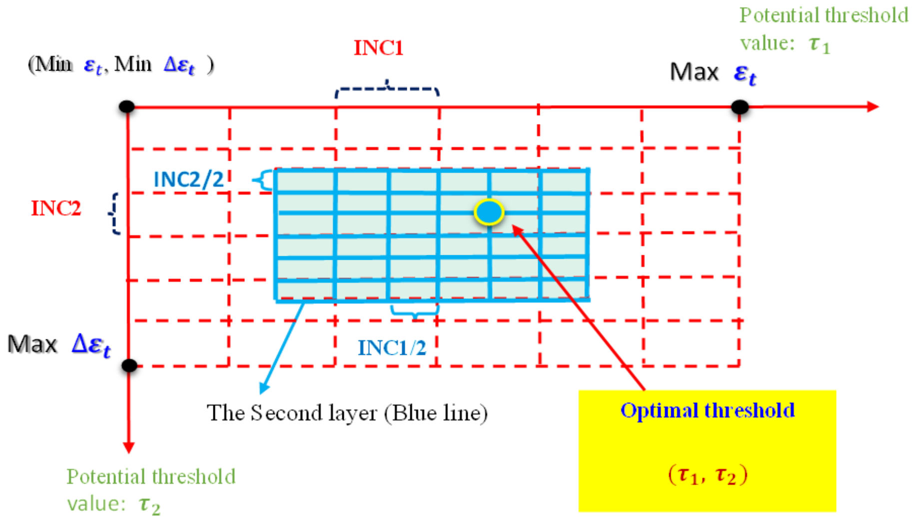

Figure 2 A searching algorithm using a two-layered loops procedure

3 | Empirical results

3.1 Examination of the asymmetric threshold effects

Panel A of Table 2 provides statistics, indicating that over this 31-year period, the 30-year U.S. Treasury futures exhibit higher average daily returns and greater volatility (measured by standard deviation) compared to the 10-year U.S. Treasury futures. Additionally, in Panel B, we observe that during periods of yield curve inversion, both the 10-year and 30-year U.S. Treasury bond futures show positive average daily returns. These returns are notably higher than the average daily returns observed across the entire 31-year sample period.

Table 2 Descriptive statistics of U.S. Treasury futures and Treasury yields in each segmented period

Panel A:		**Mean**	**Std.Dev.**	**Maximum**	**Minimum**
Entire sample period: 1990.06.26~2023.12.29	10Y T-note futures price	116.2443	10.5610	140.3312	93.1880
	10Y T-note futures return	0.0032%	0.3782%	3.5375%	-2.6277%
	30Y T-Bond futures price	125.1721	21.5744	186.9508	87.2220
	30Y T-Bond futures return	0.0045%	0.6211%	3.9698%	-4.1085%
Panel B:		Mean	Std.Dev.	Maximum	Minimum
Inversion of the 10-year and 2-year Treasury note yields	10Y T-note futures price	107.8113	6.5085	131.3543	94.4020
	10Y T-note futures return	0.0045%	0.3890%	1.9753%	-1.1311%
	30Y T-Bond futures price	114.8445	13.4581	166.9170	92.8764
	30Y T-Bond futures return	0.0050%	0.6469%	3.6682%	-2.4960%
Panel C:		Mean	Std.Dev.	Maximum	Minimum
Entire sample period: 1990.06.26~2023.12.29	2-Year Treasury Note Yield	3.1000%	2.2496%	8.4130%	0.1013%
	10-Year Treasury Note Yield	4.1779%	1.9416%	9.0360%	0.5069%
	30-Year Treasury Bond Yield	4.7094%	1.7850%	9.1720%	0.9953%
Panel D:		Mean	Std.Dev.	Maximum	Minimum
Inversion of the 10-year and 2-year Treasury note yields	2-Year Treasury Note Yield	4.9755%	0.9273%	6.9080%	1.5000%
	10-Year Treasury Note Yield	4.6429%	0.9988%	6.6390%	1.4711%
	30-Year Treasury Bond Yield	4.7102%	0.9036%	6.3420%	1.9501%

Note: we obtained daily data on 10-year and 30-year U.S. Treasury futures prices provided by CME Group from Bloomberg, and We collected the daily yields of 2-year、10-year and 30-year U.S. bonds from Bloomberg.

Furthermore, when we compare the standard deviations presented in Table 2, the data show that the standard deviations of U.S. Treasury bond futures prices and yields tend to be lower. A unit root test on daily U.S. Treasury bond futures prices indicates a common presence of unit roots in time series data, addressed by first-differencing. Additionally, a Chow test found no evidence of structural breaks in Treasury bond futures prices during yield curve inversion. To assess the effectiveness of different volatility estimation models, this study empirically examines all hypotheses using the SMTAR-GARCH model. Additionally, it conducts a concurrent comparison of volatility estimation outcomes between the GARCH model and the GJR-GARCH model.

First, we analyze the empirical results of hypotheses regarding the volatility of 10-year U.S. Treasury note futures prices during inverted yield curve periods. In Table 3, when both threshold values for testing asymmetric volatility are set to zero by default (GJR-GARCH), the estimated coefficients (ρ_1 、ρ_2) for the TAR and MTAR in the variance equation are statistically significant. However, the estimated coefficients for the interaction terms of the inverted yield curve dummy variable with the TAR effect (δ_2) and the MTAR effect (δ_3) are not statistically significant.

Table 3 Hypothesis testing on the 10-year U.S. Treasury note futures (during the period of the yield curve inversion between the 10-year and 2-year government bonds)

SMTAR-GARCH Model:

$$R_t = \mu_0 + \mu_1 R_{t-1} + \mu_2 I_t + \mu_3 M_t + \mu_4 D_t + \mu_5 D_t \cdot I_t + \mu_6 D_t \cdot M_t + \varepsilon_t$$

$$h_t = \alpha_0 + \beta_1 h_{t-1} + \alpha_1 \varepsilon_{t-1}^2 + \rho_1 I_t + \rho_2 M_t + \delta_1 D_t + \delta_2 D_t \cdot I_t + \delta_3 D_t \cdot M_t$$

Equations (5) and (6) of the SMTAR-GARCH model represent the mean equation and variance equation, respectively, for the price changes of U.S. Treasury bond futures. The variable I_t represents the threshold dummy variable for the TAR effect, and variable M_t is defined as a threshold dummy variable for the MTAR effect. Variable D_t is a dummy variable indicating whether the yield curve is inverted. D_t represents an indicator for the yield curve inversion between the 10-year and 2-year US Treasury notes.

10-year Treasury note	**GARCH(1,1)**		**GJR-GARCH**		**SMTAR-GARCH**	
Inversion of the 10-year and 2-year Treasury note yield	**(No threshold)**		**($\tau_1 = \tau_2 = 0$)**		**(τ_1 = 0.3146; τ_2 = 0.3908)**	
	Coeff.	**t-Stat**	**Coeff.**	**t-Stat**	**Coeff.**	**t-Stat**
Mean equation						
μ_0	0.0104	2.0553***	-0.0060	-0.6373	0.0067	0.8955
μ_1	-0.0077	-0.5999	-0.0092	-0.6762	0.0020	0.1060
μ_2			0.0212	0.7675	-0.0013	0.3263
μ_3			0.0224	1.5028	-0.0142	-0.7779
μ_4	0.0035	0.2763	-0.0063	-0.4954	0.0075	0.4288
μ_5			-0.0003	-0.0078	0.0091	0.7086
μ_6			-0.0260	-0.6692	0.0152	0.8231
Variance equation						
α_0	0.0085	16.5197***	0.3811	85.3179***	0.0063	9.6374***
β_1	0.9070	156.376***	-0.8518	-51.0150***	0.8831	165.0639***
α_1	0.0584	12.9004***	0.0024	9.1668***	0.0534	12.4493***

Table 3 Hypothesis testing on the 10-year U.S. Treasury note futures (continued)

10-year Treasury note	**GARCH(1,1)**		**GJR-GARCH**		**SMTAR-GARCH**	
Inversion of the 10-year and 2-year Treasury note yield	**(No threshold)**		**($\tau_1 = \tau_2 = 0$)**		**(τ_1 = 0.3146; τ_2 = 0.3908)**	
	Coeff.	**t-Stat**	**Coeff.**	**t-Stat**	**Coeff.**	**t-Stat**
Variance equation						
ρ_1			-0.0985	-13.0892***	-0.0157	-7.9694***
ρ_2			-0.0226	-7.1981***	0.0477	37.3874***
δ_1	-0.0058	-11.7101***	0.0419	14.0776***	-0.0035	-4.5331***
δ_2			0.0094	1.3885	-0.0035	4.7082***
δ_3			-0.0433	-7.8359***	-0.0035	-5.5619***
AIC	-5660.1767		-5153.7006		-5881.2807	
SBC	-5665.1054		-5174.6626		-5902.2385	
Log-likelihood	2837.0883		2591.8503		2955.6404	
Likelihood ratio test					LR/ p-value	
(H_0: $\tau_1 = \tau_2 = 0$)					728/0.0***	
Ljung–Box test	Q-stat	p-value	Q-stat	p-value	Q-stat	p-value
Lags 1	1.060	0.3033	0.001	0.9169	1.006	0.8636
Lags 5	10.108	0.0722*	10.035	0.0742*	6.861	0.2312
Lags 15	25.425	0.0445**	24.616	0.0553*	21.826	0.1124

Note: a Left-tailed test. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively. Akaike Information Criterion (AIC) and Schwarz's Bayesian Criterion (SBC) are both statistical measures that are used in statistical model selection to identify the best-fitting model for a given set of data. A lower AIC or SBC value indicates a better model fit.

Within the SMTAR-GARCH model, the dual thresholds for capturing asymmetric volatility are determined as random values within a specific range. These optimal threshold values were identified through the implementation of our grid search algorithm. It was found that when π_1 =0.3146 and π_2 =0.3908, the estimated coefficients associated with the interaction terms between the inverted yield curve dummy variable and both the TAR effect and the MTAR effect become statistically significant. Additionally, it is noteworthy that both the Akaike Information Criterion (AIC) and Schwarz's Bayesian Criterion (SBC) consistently indicated that the SMTAR-GARCH model represents the best-fitting model for the given dataset under consideration. Furthermore, the results of the likelihood ratio test also demonstrate that the SMTAR-GARCH model has a better fit than the GJR-GARCH model.

Table 3 results for H_1 show a significant negative coefficient δ_1, indicating a decrease in price volatility of 10-year U.S. Treasury futures during yield curve inversion periods. This supports our H_1. The coefficient (δ_2) of the interaction term for the TAR effect during yield curve inversion is both statistically significant and negative. This indicates a threshold effect on the price volatility of 10-year Treasury futures during this period. Specifically, when facing negative news shocks impacting bond market returns, volatility tends to increase, but only when the negative shock exceeds a predefined threshold level. This result supports H$_2$. In contrast, when positive news dominates the bond market, and its positive impact surpasses the predefined threshold level, the TAR effect is activated, leading to a significant reduction in the volatility of bond futures prices. This result supports H$_2$.

The coefficient δ_3 for the MTAR effect is statistically significant and negative. This indicates a threshold effect on bond futures price volatility when negative news shocks occur in the bond market during this period. Essentially, significant changes in shock magnitude or lagged residual (first-differencing) falling below its momentum threshold lead to increased volatility. Conversely, positive news in the bond market, surpassing the threshold, decreases bond futures price volatility. This supports H$_3$.

Table 4 reaffirms SMTAR-GARCH's superior fit over GARCH and GJR-GARCH models. It also shows reduced volatility in 30-year U.S. Treasury futures during inverted yield curve periods, supporting H$_1$. Statistically significant coefficients (δ_2) and (δ_3)for TAR and MTAR effects endorse H$_2$ and H$_3$.

Table 4 Hypothesis testing on the 30-year U.S. Treasury bond futures (during the period of the yield curve inversion between the 10-year and 2-year government bonds)

SMTAR-GARCH Model:

$$R_t \;=\; \mu_0 \;+\mu_1 R_{t-1} \;+\mu_2 I_t \;\;+\mu_3 M_t \;\;+\mu_4 D_t + \mu_5 D_t \cdot I_t \;+\mu_6 D_t \cdot M_t \;\;+\varepsilon_t$$

$$h_t \;=\; \alpha_0 + \beta_1\, h_{t-1} + \;\alpha_1\, \varepsilon_{t-1}^2 + \rho_1 I_t \;\;+\rho_2 M_t \;\;+\delta_1 D_t \;+\delta_2 D_t \;\cdot I_t \;\;+\delta_3 D_t \cdot M_t$$

Equations (5) and (6) of the SMTAR-GARCH model represent the mean equation and variance equation, respectively, for the price changes of U.S. Treasury bond futures. The variable I_t represents the threshold dummy variable for the TAR effect, and variable M_t is defined as a threshold dummy variable for the MTAR effect. Variable D_t is a dummy variable indicating whether the yield curve is inverted. D_t represents an indicator for the yield curve inversion between the 10-year and 2-year US Treasury notes.

30-year Treasury bond	**GARCH(1,1)**		**GJR-GARCH**		**SMTAR-GARCH**	
Inversion of the 10-year and 2-year Treasury note yield	**(No threshold)**		**(τ_1 = τ_2 = 0)**		**(τ_1 = -1.1230; τ_2 = 0.3209)**	
	Coeff.	**t-Stat**	**Coeff.**	**t-Stat**	**Coeff.**	**t-Stat**
			Mean equation			
μ_0	0.0139	1.4741	-0.0160	-0.9039	-0.0471	-1.1221
μ_1	-0.0195	-1.5073	-0.0325	-1.7055*	-0.0357	-1.7441*
μ_2			0.0179	0.5347	0.2522	0.5837
μ_3			-0.0021	-0.0852	0.0303	0.7387
μ_4	0.0085	0.3914	0.0431	2.0766**	-0.0002	0.0463
μ_5			0.0511	1.0487	-0.2544	-0.5831
μ_6			-0.0491	-1.0111	-0.0586	1.0425
			Variance equation			
α_0	0.0190	16.4883***	0.0297	9.4957***	0.2333	10.9262***

Table 4 Hypothesis testing on the 30-year U.S. Treasury bond futures (during the period of the yield curve inversion between the 10-year and 2-year government bonds) (continued)

30-year Treasury bond	**GARCH(1,1)**		**GJR-GARCH**		**SMTAR-GARCH**	
Inversion of the 10-year and 2-year Treasury note yield	**(No threshold)**		**($\tau_1 = \tau_2 = 0$)**		**(τ_1 = -1.1230; τ_2 = 0.3209)**	
	Coeff.	**t-Stat**	**Coeff.**	**t-Stat**	**Coeff.**	**t-Stat**
			Variance equation			
β_1	0.9264	194.7034***	0.9422	285.1159***	0.7305	54.7777***
α_1	0.0523	13.9683***	0.0452	16.8834***	0.0857	10.1147***
ρ_1			-0.0208	-2.8449***	-0.0979	-2.8712***
ρ_2			-0.0433	-15.6308***	0.0929	4.4332***
δ_1	-0.0150	-13.4983***	0.0081	1.24429	-0.0042	2.3142**
δ_2			0.0378	4.2893***	-0.0824	-2.6210***
δ_3			-0.0162	-1.3950	-0.0824	-2.1112**
AIC	4313.8981		4178.0186		3700.8959	
SBC	4308.9694		4157.0566		3679.4702	
Log-likelihood	-2149.9491		-2074.0093		-1835.4480	
Likelihood Ratio Test					LR/ *p*-value	
($H_0: \tau_1 = \tau_2 = 0$)					477/0.0***	
Ljung–Box test	Q-stat	*p*-value	Q-stat	*p*-value	Q-stat	*p*-value
Lags 1	0.330	0.5657	0.388	0.5332	0.292	0.5889
Lags 5	7.969	0.1579	3.810	0.5771	4.475	0.4832
Lags 15	27.663	0.0238**	23.494	0.0742*	22.028	0.1071

Note: a Left-tailed test. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively. Akaike Information Criterion (AIC) and Schwarz's Bayesian Criterion (SBC) are both statistical measures that are used in statistical model selection to identify the best-fitting model for a given set of data. A lower AIC or SBC value indicates a better model fit.

An important discovery from Table 4 lies in the determination of the TAR and MTAR thresholds for 30-year U.S. Treasury bond futures price volatility. Specifically, these thresholds are identified through our grid search algorithm as τ_1= -1.123 and τ_2= 0.3209, respectively. It is noteworthy that both thresholds are lower than the threshold level for the 10-year U.S. Treasury bond futures in Table 3. Moreover, τ_1 in Table 4 is negative, indicating that a more severe negative shock is required to trigger the TAR threshold effect for 30-year U.S. Treasury bond futures price volatility.

On the other hand, the smaller threshold value for τ_2 in Table 4 indicates that 30-year U.S. Treasury bond futures price volatility exhibits a higher sensitivity to variations in the magnitude of market shocks in comparison to 10-year U.S. Treasury bond futures. This lower MTAR threshold level for 30-year U.S. Treasury bond futures suggests that its asymmetric MTAR effect is more prone to occurrence when contrasted with that of 10-year U.S. Treasury note futures.

Table 5 compares the overall impacts of TAR and MTAR on U.S. Treasury bond futures price volatility. Despite findings in Tables 3 and 4 suggesting that MTAR's interaction effect during inversion periods tends to increase volatility when negative shock magnitude surpasses the threshold, the coefficient ρ_2, being a larger positive value, offsets MTAR's negative correlation effect during inversion, resulting in a positive correlation between MTAR's total effect and market impact ($\rho_2 + \delta_3 > 0$). Additionally, the total effect of TAR still maintained a negative correlation with market impact during yield curve inversion period ($\rho_1 + \delta_2 < 0$). The Wald test in Table 5 reveals that the total effect of MTAR is significantly greater than the total effect of TAR.

The hypothesis testing in this study focuses on the interaction effects of TAR and MTAR with an inverted yield curve (i.e., only partial volatility of bond futures price). To further understand the complete market impact of TAR and MTAR on bond futures volatility during periods of yield curve inversion, we utilized the News Impact Curve (NIC) of the total TAR and total MTAR effects and further derived the asymmetric volatility thresholds considering the total TAR and total MTAR effects from the NIC.

Based on the coefficient estimates obtained from equation (11), equation (12) and equation (13) represent the News Impact Curves (NICs) for the total TAR effect and total MTAR effect, respectively, during periods of yield curve inversion, i.e., $D_t = 1$.

3.2 Further discussion on the total TAR effects and the total MTAR effects

News impact curve (NIC) of total TAR effects:

$$NIC(\varepsilon_{t-1}) = \alpha_1 \varepsilon_{t-1}^2 + \rho_1(\varepsilon_{t-1} - \tau_1) + \delta_2(\varepsilon_{t-1} - \tau_1) \tag{12}$$

News impact curve of total MTAR effects:

$$NIC(\Delta\varepsilon_{t-1}) = \alpha_1(\Delta\varepsilon_{t-1} + \varepsilon_{t-2})^2 + \rho_2(\Delta\varepsilon_{t-1} - \tau_2) + \delta_3(\Delta\varepsilon_{t-1} - \tau_2) \tag{13}$$

The NIC threshold values for the total TAR effect and total MTAR effect are derived from the first-order derivatives of equation (12) and equation (13). We present the News Impact Curves (NICs) for the total TAR effect and total MTAR effect in Figure 3 and Figure 4.

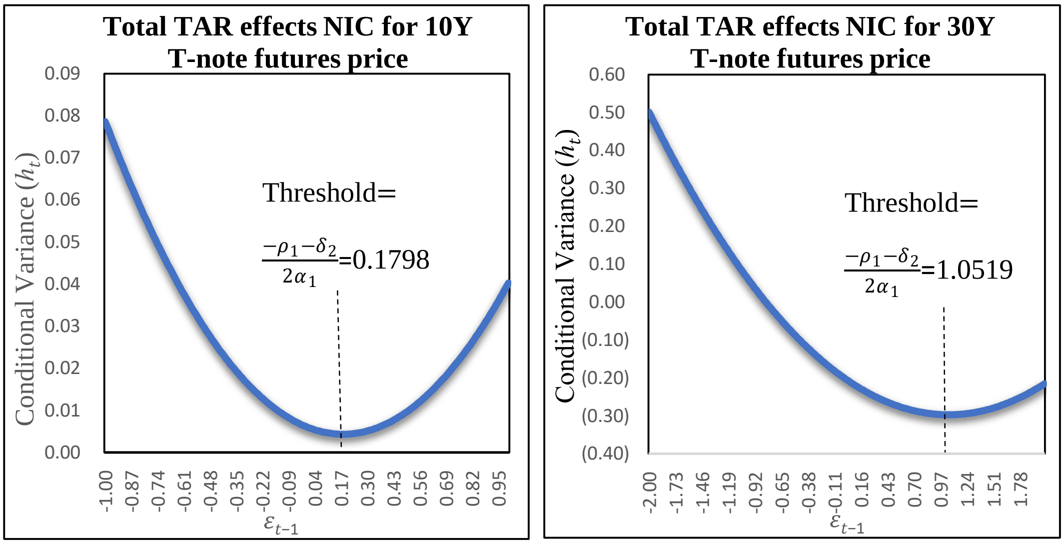

Figure 3 News impact curves for total TAR effect

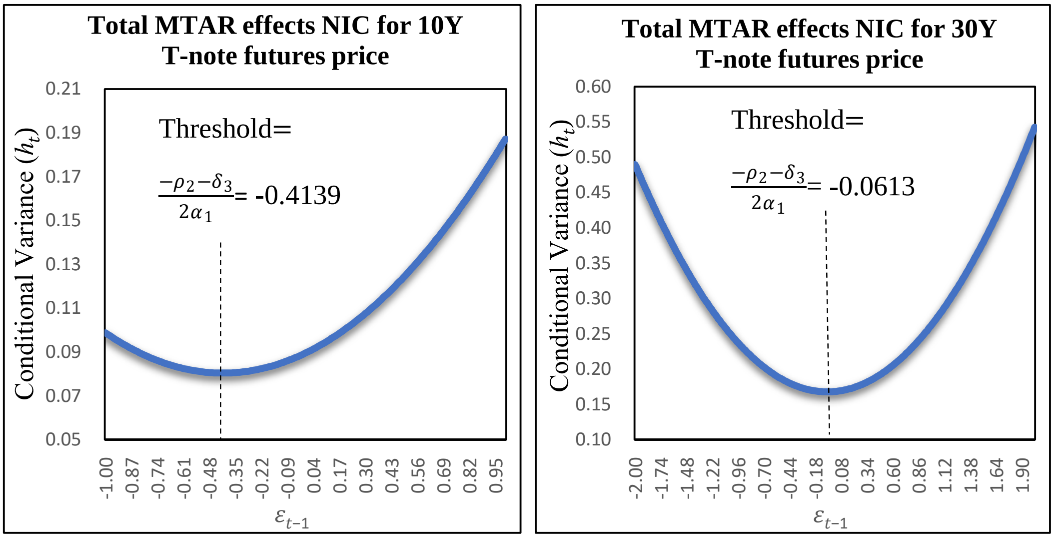

Figure 4 News impact curves for total MTAR effect

It is worth noting that the TAR and MTAR effects do not necessarily occur simultaneously. However, if both the TAR and MTAR effects are triggered during periods of yield curve inversion, the News Impact Curve (NIC) for the combined total TAR and total MTAR effects is as follows:

News impact curve of combined total TAR and total MTAR effects:

$$NIC(\varepsilon_{t-1}) = 2\alpha_1\varepsilon_{t-1}^2 + \rho_1(\varepsilon_{t-1}-\tau_1) + \delta_2(\varepsilon_{t-1}-\tau_1) + \rho_2(\varepsilon_{t-1}-\tau_2) + \delta_3(\varepsilon_{t-1}-\tau_2) \quad (14)$$

To illustrate the News Impact Curve (NIC) of the combined total TAR and total MTAR effects, we consider a special case where $\varepsilon_{t-1} = \Delta\varepsilon_{t-1} = \varepsilon_{t-1} - \varepsilon_{t-2}$, $i.e.$ $\varepsilon_{t-2} = 0$, the threshold values for the combined total TAR and total MTAR effects can also be derived from the first-order derivative of equation (14). The corresponding graphs are shown in Figure 5.

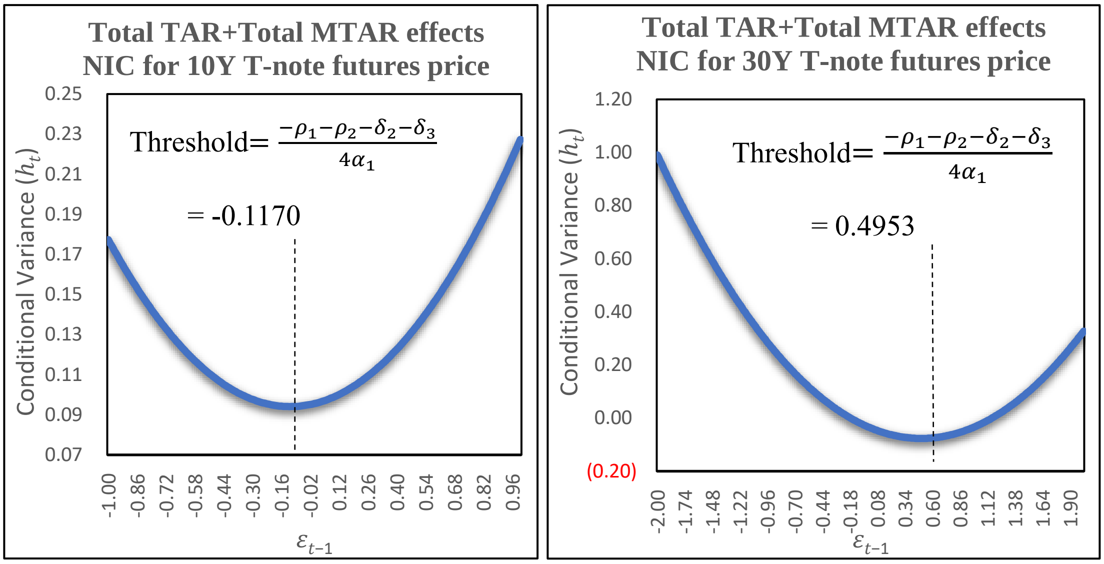

Figure 5 News Impact Curve (NIC) of the combined total TAR and total MTAR effects

3.3 Implications of asymmetric threshold volatility effects

The empirical findings of this study reveal that during periods of yield curve inversion, there is evidence of asymmetric volatility in the 10-year and 30-year U.S. Treasury bond futures. Specifically, in terms of the TAR and MTAR effects interacting with the inverted yield curve, bond futures volatility increases when negative news occurs (exceeding the threshold), while positive news leads to a decrease in volatility (as hypothesized in H_2 and H_3 of this study). The effects during the inversion (interaction effects between D_t and I_t,as well as between D_t and M_t) are illustrated in Figure 6.

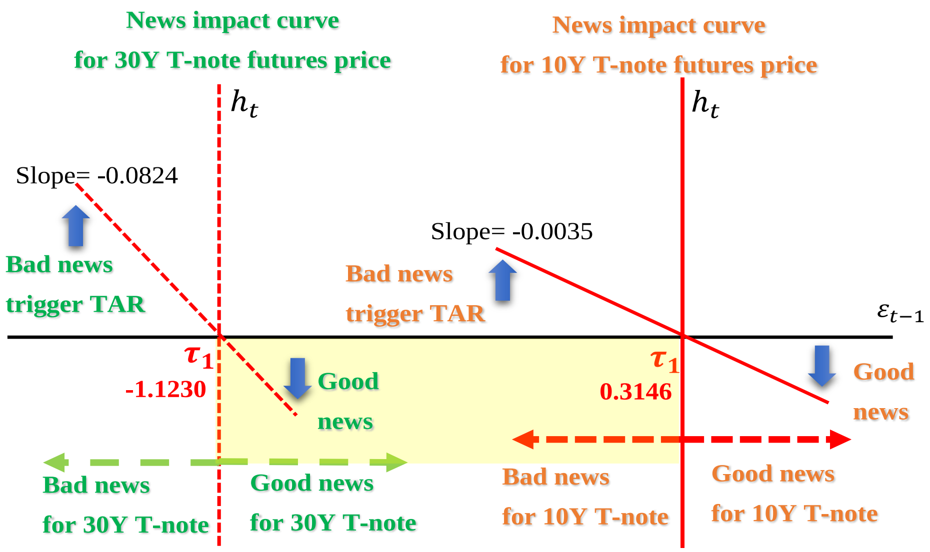

Figure 6 Schematic diagram of TAR effects during inversion

Furthermore, when considering the complete Total TAR effects, Table 5 and Figure 3 show that due to the interaction effects between D_t and I_t associated with the inverted yield curve, the Total TAR effects for 10-year and 30-year U.S. Treasury bond futures exhibit an asymmetric pattern, where volatility is higher in response to bad news and relatively lower in response to good news. The thresholds for bad and good news are determined by the SMTAR-GARCH model's estimated threshold values.

Table 5 Comparison of total effects between TAR and MTAR

All estimated coefficients in Table 5 were extracted from Tables 3 and Table 4

Smtar-Garch	**10-year Treasury note futures**	**30-year Treasury bond futures**
Dual threshold(π_1/π_2)	0.3146/ 0.3908	-1.1230/ 0.3209
D_t (δ_1)	-0.0035***	-0.0042**
TAR (ρ_1)	-0.0157***	-0.0979***
$MTAR$ (ρ_2)	0.0477***	0.0929***
$D_t * TAR$ (δ_2)	-0.0035***	-0.0824***
$D_t * MTAR$ (δ_3)	-0.0035***	-0.0824**
Total TAR Effect $(\rho_1+\delta_2)$	-0.0192	-0.1803
Total MTAR Effect $(\rho_2+\delta_3)$	0.0442	0.0105
$\rho_2+\delta_3>\rho_1+\delta_2$	χ^2/ *p*-value	χ^2/ *p*-value
	99.63135/0.0***	29.2900/0.0***

Note: a Left-tailed test. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively. χ^2 is the Wald test statistic distributed as a chi-squared distribution.

In addition, the Total MTAR effects (as shown in Table 5 and Figure 4) present results opposite to those of the Total TAR effects. When the magnitude of changes in bad news exceeds the threshold of the MTAR model, volatility is actually lower compared to the impact of strong positive news.

3.4 Implications of sensitivity of volatility to market shocks

The determination of threshold values simultaneously reflects the volatility sensitivity of bond futures. Table 6 summarizes the various threshold values for asymmetric effects discussed in this study.

Table 6 Threshold values and implications for volatility sensitivity under different effects

Threshold values	**10-year T-Note futures**	**30-year T- note futures**	**Implications for sensitivity**
TAR effects	0.3146	-1.1230	For 10Y T-note futures price, higher threshold means greater sensitivity to bad news (smaller to good news). For 30Y T-note futures price, smaller threshold means lower sensitivity to bad news (greater to good news).
MTAR effects	0.3908	0.3209	MTAR threshold for 30-year T-note futures price is lower, indicating higher sensitivity to changes of market good news.
Total TAR effects	0.1798	1.0519	The total TAR effect on 10-year and 30-year T-note futures price is more sensitive to bad news (the 30-year T-note futures price is more sensitive than 10-year T-note futures price). Regarding good news, 10-year T-note futures price is more sensitive than 30-year T-note futures price.
Total MTAR effects	-0.4139	-0.0613	Both are more sensitive to changes in the impact of positive market news, and the 30-year T-note futures price is even more sensitive than 10-year T-note futures price. Regarding bad news, 30-year T-note futures price is more sensitive than 10-year T-note futures price.
Total TAR + Total MTAR	-0.1170	0.4953	10-year T-note futures price is more sensitive to positive market news. 30-year T-note futures price is more sensitive to negative market news.

Note: TAR effects are interaction effects between D_t and I_t, MTAR effects are interaction effects between D_t and M_t.

First, regarding the TAR effect, the optimal threshold values determined using the grid search algorithm indicate that the threshold value for the 10-year U.S. Treasury futures TAR effect is 0.3146, which is higher than the threshold value of -1.1230 for the 30-year U.S. Treasury futures. For the 10-year U.S. Treasury futures, a higher threshold value (positive) signifies greater sensitivity to bad news, meaning the worse-than-expected good news could become bad news and amplify bond futures price volatility. For the 30-year U.S. Treasury futures, the TAR effect threshold is lower and negative (-1.123), indicating that the 30-year U.S. Treasury futures are more sensitive to positive market news, where better-than-expected bad news can be perceived as good news.

Additionally, for the MTAR effect, MTAR threshold for 30-year U.S. Treasury bond futures is lower compared to that of 10-year U.S. Treasury bond futures, indicating increased sensitivity to market shock changes during this period. Minor fluctuations in market conditions can activate MTAR threshold effects.

Moreover, when considering the comprehensive Total TAR effect, Table 6 and Figure 3 show that the threshold values for the Total TAR effect on 10-year and 30-year U.S. Treasury futures are both positive (0.1798 and 1.0519, respectively), with the threshold for the 30-year U.S. Treasury futures being higher than that for the 10-year U.S. Treasury futures. This indicates that the Total TAR effect for both the 10-year and 30-year U.S. Treasury futures is more sensitive to negative market news (since the threshold values are positive), and that the 30-year U.S. Treasury futures are more sensitive to bad news than the 10-year U.S. Treasury futures. Conversely, regarding positive market news, the Total TAR effect for the 10-year U.S. Treasury futures is more sensitive than that for the 30-year U.S. Treasury futures.

Furthermore, since the threshold values for the Total MTAR effect for the 10-year and 30-year U.S. Treasury futures have turned negative, both are more sensitive to changes in the magnitude of positive market news, with the 10-year U.S. Treasury futures being even more sensitive due to its smaller threshold value. However, in terms of negative market news, the Total MTAR effect for the 30-year U.S. Treasury futures is more sensitive.

Finally, discussing the special case in Figure 5 where both the Total TAR and Total MTAR effects occur simultaneously, the threshold for the 10-year U.S. Treasury futures is negative (-0.1170), making it more sensitive to positive market news. In contrast, the threshold for the 30-year U.S. Treasury futures is positive (0.4953), indicating greater sensitivity to negative market news. All threshold values and implications of asymmetrical effects are summarized in Table 6.

3.5 Economic implications and practical applications of trading strategies

Based on our empirical findings, this study emphasizes the significant economic implications of asymmetric volatility thresholds in bond futures. These insights provide valuable guidance for pricing financial instruments or formulating trading strategies linked to bond volatility, such as bond futures options.

From Table 7 and Table 8, we infer the economic significance and implications for trading strategies. Specifically, during yield curve inversion periods, due to the presence of asymmetric threshold effects, for buyers of bond futures options, regardless of considering total TAR or total MTAR effects, the optimal trading strategy is to purchase 30-year Treasury futures put options and 10-year Treasury futures call options, leveraging different market impacts and volatility sensitivities to influence trading strategy outcomes. However, for sellers of Treasury futures options, under consideration of the total TAR effect, the implied preferable strategy is to sell 30-year Treasury futures call options, while under consideration of the total MTAR effect, the implied preferable strategy is to sell 10-year Treasury futures put option.

Table 7 Implications for bond futures options trading strategies under consideration of total TAR effects

Implications for bond futures options trading strategies	**Under consideration of total TAR effects**	**Potential sources of profit for trading strategies**
Options buyers	Put on 30Y-T note futures	The increasing vega of the option (30Y-T note Futures are more sensitive to bad news)
	Call on 10Y-T note futures	The increasing vega of the option (Regarding good news, 10-year T-note futures are more sensitive than 30-year T-note futures.)
Options sellers	Call on 30Y-T note futures	The decreasing vega of the option (30Y-T note futures are relatively insensitive to good news)

Note: Vega is the Greek that measures the amount of increase or decrease in an option premium based on a 1% change in implied volatility.

Table 8 Implications for bond futures options trading strategies under consideration of total MTAR effects

Implications for bond futures options trading strategies	**Under consideration of total MTAR effects**	**Potential sources of profit for trading strategies**
Options buyers	Put on 30Y-T note futures	The increasing vega of the option (Regarding bad news, 30Y-T note futures are more sensitive than 10-year T-note futures
	Call on 10Y-T note futures	The increasing vega of the option (10-year T-note futures are more sensitive to good news)
Options sellers	Put on 10Y-T note futures	The decreasing vega of the option (10Y-T note futures are relatively insensitive to bad news)

Note: Vega is the Greek that measures the amount of increase or decrease in an option premium based on a 1% change in implied volatility.

4 | Economic significance and conclusions

Unlike most literature focused on financial product volatility during financial crises, this study evaluates bond futures price volatility in the context of yield curve inversion in the bond market. Moreover, the literature suggests that the bond market exhibits asymmetric volatility in response to market shocks. This study further examines whether this asymmetric volatility persists during periods of yield curve inversion, where the 10-year U.S. Treasury yield is lower than the 2-year U.S. Treasury yield. It also investigates how the volatility of 10-year and 30-year bond futures prices reacts to positive and negative market shocks. Additionally, this study compares the sensitivity of these bond futures to market shocks.

We introduce the SMTAR-GARCH model, efficiently identifying TAR and MTAR effects with their random threshold values. We determine optimal thresholds through a grid search using a loop algorithm. Utilizing daily trading data of 10-year and 30-year U.S. Treasury futures prices spanning from 1990 to 2023, the findings of this research indicate a reduction in the volatility of U.S. Treasury bond futures prices during inverted yield curve periods.

The findings of this study can be divided into two key aspects. First, it identifies the phenomenon of asymmetric volatility in bond futures. The empirical results reveal that U.S. medium- to long-term Treasury bond futures exhibit a threshold effect of asymmetric volatility during periods of yield curve inversion. The economic implication is that the volatility response of bond futures to market shocks, whether good news or bad news, is not uniform. This allows for a comparison of the volatility differences in bond futures when responding to positive and negative market shocks.

According to the empirical results of this paper, the Total TAR effects indicate that when the magnitude of negative market news exceeds the threshold, bond futures volatility increases significantly. Meanwhile, the Total MTAR effects demonstrate that when the magnitude of positive market news changes exceeds the threshold, bond futures volatility also increases substantially. This finding is valuable for developing bond options trading strategies that are highly sensitive to volatility, such as determining whether an options buyer should purchase put or call options on bond futures.

If investors expect market shocks to reflect only Total TAR effects, the optimal trading strategy would be to purchase U.S. Treasury futures put options. This is because, in the event of negative market news, bond futures volatility will rise sharply, increasing the market value of put options. On the other hand, if investors anticipate severe market shocks that could trigger Total MTAR effects, the best strategy would be to buy bond futures call options. In this scenario, significant positive market news would lead to a substantial increase in bond futures volatility, thereby boosting the value of bond futures call options.

Secondly, the second major finding of this paper is the sensitivity analysis of bond futures volatility to market shocks. Additionally, the comparison of volatility sensitivity between 10-year and 30-year U.S. Treasury bond futures in response to market shocks represents an area that has not been explored in previous literature.

The TAR and MTAR threshold values derived from the simulations using the model in this study are economically meaningful.

The size of the threshold determines whether a market shock is classified as good news or bad news. Therefore, the level of the threshold can represent the sensitivity to market volatility. The comparison of volatility sensitivity can be analyzed on two levels:

A. The level of the threshold values in TAR or MTAR models itself represents the sensitivity of bond futures to market impacts, indicating whether the response to market shocks is perceived as good or bad news. As illustrated in Figure 7, when the threshold value of the Total TAR/MTAR effect is greater than zero, it signifies that less favorable market news tends to generate adverse reactions (i.e., higher sensitivity to bad news). Conversely, if the threshold value is less than zero, it indicates that less negative market news tends to generate positive reactions (i.e., higher sensitivity to good news).

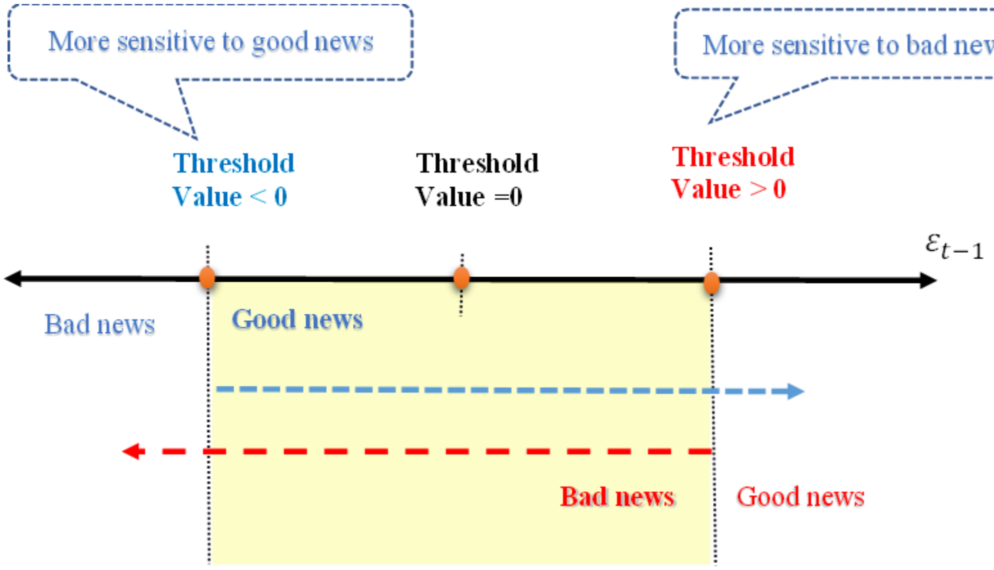

Figure 7 Economic implications of the threshold values for total TAR effects and rotal MTAR effects

B. Secondly, comparing the differences in threshold values between different bond futures can reflect the sensitivity of various futures to market shocks, either as good or bad news. In Figure 8, since the threshold values of both Total TAR effects and Total MTAR effects for the 30-year Treasury futures are higher than those for the 10-year Treasury futures, it indicates that the 10-year U.S. Treasury futures are relatively more sensitive to positive market news, while the 30-year U.S. treasury futures are more sensitive to negative news. The yellow area in Figure 8 represents the region where a market shock would be perceived as positive news for the 10-year U.S. treasury futures but as negative news for the 30-year U.S. treasury futures.

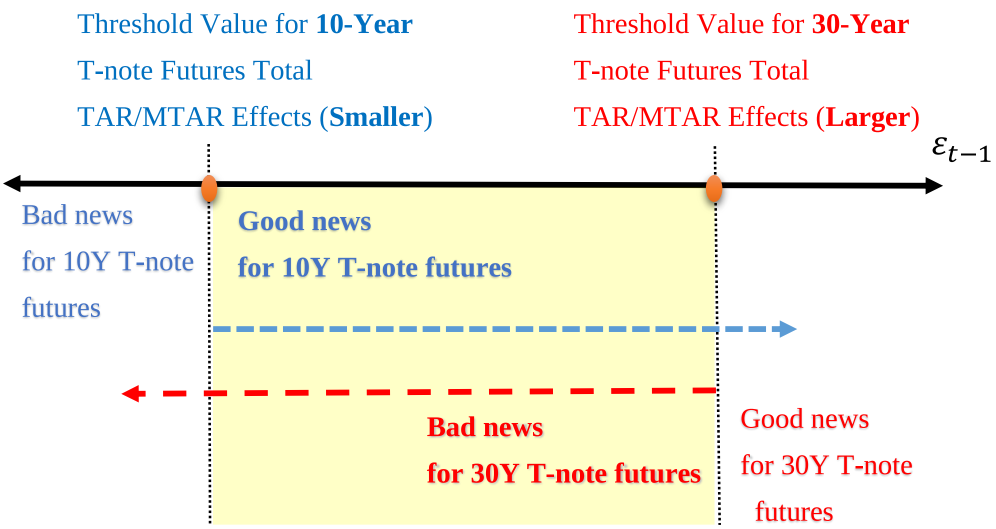

Figure 8 Comparison of sensitivity to market shocks between different maturities of treasury futures based on total TAR effects and total MTAR effects

The economic implication of the volatility sensitivity comparison analysis lies in understanding how bond futures of different maturities react to positive and negative market news. This insight is valuable for selecting the optimal underlying asset in options trading strategies. For example, when considering a put option strategy based on Total TAR effects, investors should opt for a 30-year U.S. Treasury bond futures put option rather than a 10-year bond futures put option. This is because the Total TAR effects threshold for the 30-year bond futures is higher than that for the 10-year bond futures, indicating that the 30-year bond futures are more sensitive to negative market news and are more likely to experience a significant increase in volatility, thereby enhancing the value of the put options.

Similarly, when considering a strategy of buying call options based on Total MTAR effects, the 10-year Treasury bond futures call option should be selected over the 30-year bond futures call option. This is primarily because the total MTAR effects threshold for the 10-year bond futures is lower than that for the 30-year bond futures, indicating that the 10-year bond futures are more sensitive to positive market news. As a result, they are more likely to experience an increase in volatility, thereby enhancing the value of the call options.

Furthermore, for options sellers, the trading strategy would be the opposite. Options sellers should avoid situations where the value of the options increases (i.e., where the volatility of the underlying bond futures rises) to prevent potential losses. Therefore, they should opt for trading strategies involving lower volatility and reduced sensitivity. In the context of Total TAR effects, the optimal strategy would be to sell 30-year Treasury bond futures call options. This is because, although volatility increases significantly in response to negative news, bond futures prices typically decline, leading to a decrease in call option prices, which benefits the option seller. Conversely, when positive news occurs, the increase in volatility is less pronounced (due to the asymmetric effect of total TAR) and the 30-year bond futures have a higher total TAR effects threshold. This means that the sensitivity to positive market news is lower, reducing the likelihood of a significant rise in call option value and thus minimizing the risk of loss for the option seller.

The primary contribution of this research lies in its pioneering empirical investigation into the asymmetric threshold effects of volatility within the context of U.S. medium- to long-term bond futures prices during periods of inverted yield curve. We anticipate that the empirical findings presented in this paper will offer valuable reference insights for academia and contribute to enhancing the ability of financial markets to accurately price a wide array of bond derivative products. Furthermore, these findings can assist in the formulation of effective trading and hedging strategies grounded in a comprehensive understanding of bond futures price volatility dynamics.

Future studies could delve into whether asymmetric threshold effects exist in short-term US Treasury futures and verify the significance and effectiveness of the trading strategies inferred in this research. Additionally, as indicators for identifying an inverted yield curve, it is recommended to incorporate the spread between the 10-year US Treasury bonds and 3-month Treasury bills, or the near-term forward spread - a new yield curve inversion indicator that has received increasing attention in the market. This approach allows for the observation of various behavioral responses in bond futures price volatility, providing richer information for both academia and financial practitioners.

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波動率之謎:探索美國政府公債期貨價格於殖利率倒掛期間之不對稱波動門檻效果

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摘要

有別於多數文獻探討金融商品於金融危機期間的波動率特性，本文為首次針對殖利率倒掛期間，檢驗美國中長期公債期貨是否具有不對稱波動門檻效果。本研究透過創新的實證模型及網格演算法找出最適隨機門檻估計值，並取得顯著的證據顯示美國中長期公債期貨於殖利率倒掛期間具有 TAR (Threshold AutoRegressive) and MTAR (Momentum Threshold AutoRegressive)門檻效果。其中 TAR 為衡量市場衝擊之水準程度;而 MTAR 衡量市場衝擊變動程度之門檻效應。

本文實證結果顯示，美國公債期貨於殖利率倒掛期間價格波動率將下降。此外，不對稱波動門檻效果顯示，當市場出現負面衝擊且程度超過門檻水準時，TAR、MTAR 效果與倒掛殖利率的交互作用將使債券期貨價格波動率上升。再者，當考量債券期貨價格完整的 Total TAR 效果時，若市場出現不利衝擊且程度超過門檻水準，債券期貨價格波動率仍會有顯著增加現象。然而當考量 Total MTAR 效果時，實證結果發現其與市場衝擊的相關性出現反轉，亦即當市場出現正面消息且衝擊變動程度大於門檻水準時，則債券期貨價格波動率出現大幅上升現象。因此對於產生不對稱波動效果的門檻水準將具有顯著的經濟意義，而各公債期貨透過實證模型所估計之最適門檻值將代表其對市場衝擊的敏感性。經由本文的敏感性比較分析，得有效應用於債券期貨選擇權的交易策略。其中對市場衝擊的波動不對稱特性得應用於債券選擇權 Call 或 Put 的選擇，而各期貨對市場衝擊的敏感性分析有助於選擇權 Underlying Asset 的決策。因此根據本文實證結果，於殖利率倒掛期間，最適選擇權交易策略為買進 30 年期美國公債期貨賣權(Put)或買進 10 年期美國公債期貨買權(Call)。

關鍵字:

倒掛殖利率曲線、債券期貨不對稱波動、債券期貨波動敏感性、SMTAR-GARCH 模型